Ferrite Z vs SWR as measured in test fixture with 50 ohm shunt resistor

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Zo = Fixture shunt resistor (50 ohms)
Z = Effective resistive impedance across bridge = RL in parallel with Zo
RL = Resistance across Zo giving a specific SWR value
XL = Reactance across Zo giving a specific SWR value
CR = Coeff of reflection
SWR = SWR value measured.

CASE WHERE RL IS RESISTIVE
SWR calculation with Zo in parallel with RL

$$SWR = \frac{1 + |CR|}{1 - |CR|}$$

Eq 1 SWR vs CR (coeff of reflection)

$$CR = \frac{Z - Zo}{Z + Zo}$$

Eq 2 CR vs Z (unknown impedance) and Zo (ref. impedance = 50)

$$Z = \frac{RL \cdot Zo}{RL + Zo}$$

Eq 3 The effective Z consists of RL (unknown) and Zo in parallel

$$SWR = \frac{1 + |CR|}{1 - |CR|}$$

Eq 1 repeated

$$SWR = \frac{\left(1 + \frac{|Z - Zo|}{|Z + Zo|}\right)}{\left(1 - \frac{|Z - Zo|}{|Z + Zo|}\right)}$$

Eq 4: substitute Eq 2 into Eq 1

$$SWR = \frac{\left[1 + \frac{\left|RL \cdot \frac{Zo}{(RL + Zo)} - Zo\right|}{\left|RL \cdot \frac{Zo}{(RL + Zo)} + Zo\right|}\right]}{\left[1 - \frac{\left|RL \cdot \frac{Zo}{(RL + Zo)} - Zo\right|}{\left|RL \cdot \frac{Zo}{(RL + Zo)} + Zo\right|}\right]}$$

Eq 5: sub. Eq 3 into Eq 4

$$SWR = \frac{-\left[Zo \cdot (2 \cdot RL + Zo) + Zo^2\right]}{\left[-Zo \cdot (2 \cdot RL + Zo) + Zo^2\right]}$$

Eq 6 Simplify Eq 5

$$RL = \frac{Zo}{(SWR - 1)}$$

Eq 7 Solve for RL in Eq 6

CASE WHERE RL IS REACTIVE = XL
SWR calculation with Zo in parallel with XL

$$SWR = \frac{1 + |CR|}{1 - |CR|}$$ Eq 1 SWR vs CR (coeff of reflection)

$$CR = \frac{Z - Zo}{(Z + Zo)}$$ Eq 2 CR vs Z (unknown impedance) and Zo (ref. impedance = 50)

$$Z = \frac{j \cdot XL \cdot Zo}{j \cdot XL + Zo}$$ Eq 8 The effective Z consists of XL (unknown) and Zo in parallel

$$CR = \frac{Z - Zo}{(Z + Zo)}$$ Eq 2 repeated

$$CR = \frac{\left[i \cdot XL \cdot \frac{Zo}{(i \cdot XL + Zo)} - Zo \right]}{\left[i \cdot XL \cdot \frac{Zo}{(i \cdot XL + Zo)} + Zo \right]}$$ Eq 9: Eq 8 into Eq 2

$$CR = \frac{-Zo}{(2i \cdot XL + Zo)}$$ Eq 10: Eq 9 simplified

$$SWR = \frac{1 + |CR|}{1 - |CR|}$$ Eq 1 repeated

$$SWR = \frac{\left(1 + \frac{|Zo|}{\sqrt{Zo^2 + 4 \cdot XL^2}} \right)}{\left(1 - \frac{|Zo|}{\sqrt{Zo^2 + 4 \cdot XL^2}} \right)}$$ Eq 11: Eq 10 into Eq 1

$$SWR = \frac{\left(\sqrt{Zo^2 + 4 \cdot XL^2} + |Zo| \right)}{\left(\sqrt{Zo^2 + 4 \cdot XL^2} - |Zo| \right)}$$ Eq 12: Eq 11 simplified

$$\begin{bmatrix} \frac{1}{2} \cdot \sqrt{-Zo^2 + \frac{(-SWR \cdot |Zo| - |Zo|)^2}{(SWR - 1)^2}} \\ \frac{-1}{2} \cdot \sqrt{-Zo^2 + \frac{(-SWR \cdot |Zo| - |Zo|)^2}{(SWR - 1)^2}} \end{bmatrix}$$

Solving for XL:
We use the positive root

$$XL = \frac{1}{2}\cdot\sqrt{-Zo^2 + \frac{(-SWR\cdot Zo - Zo)^2}{(SWR-1)^2}}$$ Eq 13

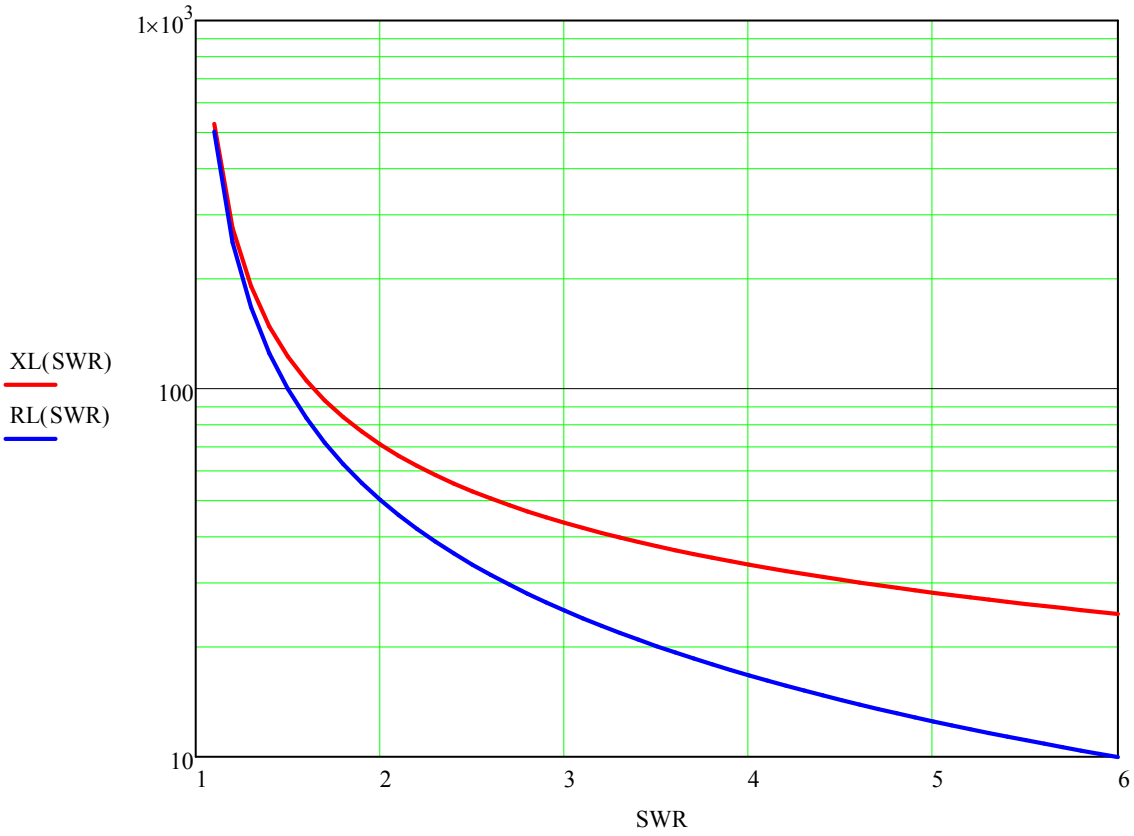
$$XL = \frac{4\cdot SWR\cdot Zo^2}{(SWR-1)^2}$$ Eq 14: Eq 13 simplified

CALCULATIONS

$Zo := 50$ $SWR := 1.1, 1.2 .. 6$

$$XL(SWR) := \frac{1}{2}\cdot\sqrt{\frac{4\cdot SWR\cdot Zo^2}{(SWR-1)^2}}$$ Eq 14

$$RL(SWR) := \frac{Zo}{(SWR-1)}$$ Eq 7



MORE CHECKS

$X := 50$ Resistance across 50 ohm termination

$Z := \frac{50 \cdot X}{50 + X}$ Eq 1 $Z = 25$ $|Z| = 25$

$CR := \frac{Z - 50}{Z + 50}$ Eq 2 $CR = -0.333$

$SWR := \frac{1 + |CR|}{1 - |CR|}$ Eq 3 $SWR = 2$

$X := j \cdot 70$ Reactance across 50 ohm termination

$Z := \frac{50 \cdot X}{50 + X}$ Eq 1 $Z = 33.108 + 23.649i$ $|Z| = 40.687$

$CR := \frac{Z - 50}{Z + 50}$ Eq 2 $CR = -0.113 + 0.317i$

$SWR := \frac{1 + |CR|}{1 - |CR|}$ Eq 3 $SWR = 2.014$