

# Q Factor Measurements on L-C Circuits

*The author reviews existing measurement techniques and offers insight into loaded and unloaded Q factors as applied to LC circuits and antennas. A simpler method is proposed that uses an SWR analyzer along with a spreadsheet that easily computes the unloaded Q.*

## Introduction

The  $Q$  factor gives a figure of merit for inductors and capacitors. It is the ratio of reactance to resistance. For filters, it relates directly to the circuit selectivity: The higher the  $Q$ , the better the selectivity and the lower the insertion loss of the filter. For oscillators, higher  $Q$  also means that lower phase noise is produced. In the case of antennas, a lower  $Q$  is generally preferred, giving a larger SWR bandwidth.

Transmission methods are traditionally used for making quality factor ( $Q$ ) measurements on L-C circuits. This implies that a signal source and an RF voltmeter or spectrum analyzer must be available for such measurements. These are not always available however. Since SWR analyzers are becoming commonplace in many amateur radio rooms, it then becomes tempting to use this instrument for  $Q$  measurements on L-C circuits.

Let's review the existing methods that are currently used for  $Q$  measurements that only require scalar measurements — that is, no phase measurements are required. The last method details how the SWR analyzer can be used to measure the unloaded  $Q$  of L-C circuits.

## 1 – The Classical Q Meter Method

This is the technique you would use if you had access to a  $Q$  Meter, such as an HP / Agilent  $Q$  Meter model HP 4342A. See Figure 1. A very low impedance source is required, typically 1 milliohm, and a very high impedance detector is connected across the L-C circuit. The unloaded  $Q$  ( $Q_U$ ) of

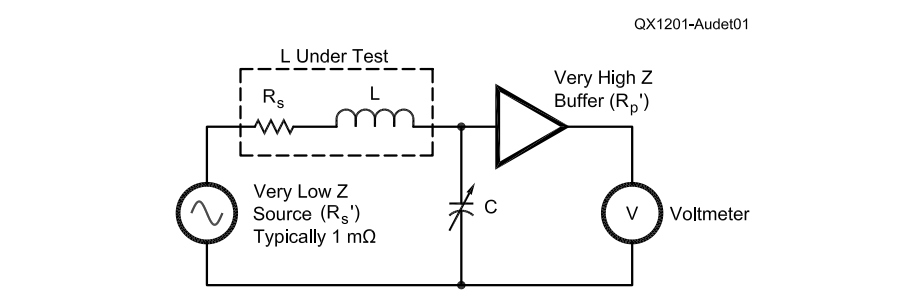


Figure 1 — Block diagram of the classical  $Q$  meter as used in the HP 4342A.

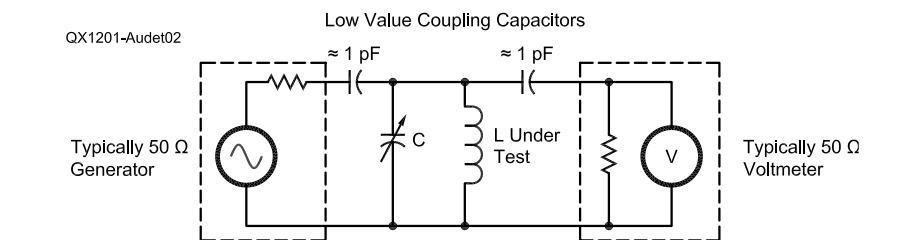


Figure 2 — The signal generator and the source are weakly coupled to the L-C circuit under test, allowing measurement of the 3 dB bandwidth.

a single reactance component is given by Equation 1.

$$Q_U = \frac{X_S}{R_S} = \frac{R_P}{X_P} \quad [\text{Eq 1}]$$

where  $X_S$  and  $R_S$  are the series reactance and loss resistance, and  $R_P$  and  $X_P$  are the corresponding parallel loss resistance and reactance components.

In the test set-up we need to make the

source resistance  $R_S'$  as small as possible, since it adds to the coil or capacitor resistance. Similarly, the detector resistance,  $R_P'$  shunting the L-C should be much higher than the  $R_P$  of the component under test. Note that the L-C circuit is commonly represented as having a resistor in series ( $R_S$ ) with L and C or a shunt resistor ( $R_P$ ) in the parallel model to represent the losses.

Measurement consists of setting the source frequency and adjusting the tuning

<sup>1</sup>Notes appear on page 11.

capacitor, C, for resonance to maximize the voltage across the resonating capacitor C.

The  $Q$  is calculated using Equation 2.

$$Q = \frac{\text{Voltage Across } C}{\text{Source Voltage}} \quad [\text{Eq } 2]$$

Note that the source voltage is in the millivolt range, since it will be multiplied by the  $Q$  factor. A 10 mV source and a  $Q$  of 500 will give 5 V across the L-C circuit. In order to preserve the high impedance of the detector even at the higher frequencies, a capacitive voltage divider is used.

This circuit measures the unloaded  $Q$  called  $Q_U$  provided that the series resistance  $R_S$  of the inductor under test is much higher than the source resistance  $R_S'$  and the detector parallel resistance  $R_P'$  is much larger than the L-C circuit  $R_P$ .<sup>1</sup>

## 2 – Transmission Method Using Coupling Capacitors

A signal is coupled into an L-C parallel circuit using a low value capacitor and extracts the output signal using the same value of capacitor.<sup>2</sup> See Figure 2. Note that inductive coupling is also possible, as used in transmission line (cavity) resonators.

The -3 dB bandwidth ( $BW$ ) is measured at the resonant frequency  $f_0$ , and the loaded  $Q$  ( $Q_L$ ) is calculated with Equation 3.

$$Q_L = \frac{f_0}{BW} \quad [\text{Eq } 3]$$

The obtained bandwidth,  $BW$ , is a function of the coupling, and another calculation is required to get  $Q_U$ , the unloaded  $Q$ . When the input to output coupling is identical, we use Equation 4.

$$\text{Loss} = 20 \log \left( \frac{Q_U}{Q_U - Q_L} \right) \quad [\text{Eq } 4]$$

where  $\text{Loss}$  is in positive dB. The ratio  $Q_U / Q_L$  may now be calculated using Equation 5.

$$\frac{Q_U}{Q_L} = \left( \frac{10^{\frac{\text{Loss}}{20}}}{10^{\frac{\text{Loss}}{20}} - 1} \right) \quad [\text{Eq } 5]$$

This equation is plotted in Figure 3.

Note that this method applies well to transmission line resonators with equal impedance inductive coupling loops at the input / output. Note also that these two methods do not require knowledge of the L or C values to compute the  $Q$  factors.

## 3 – Shunt Mode Transmission Method With L-C in Series

Here the L and C are series connected and placed in shunt across the transmission circuit. See Figure 4. (This method is also

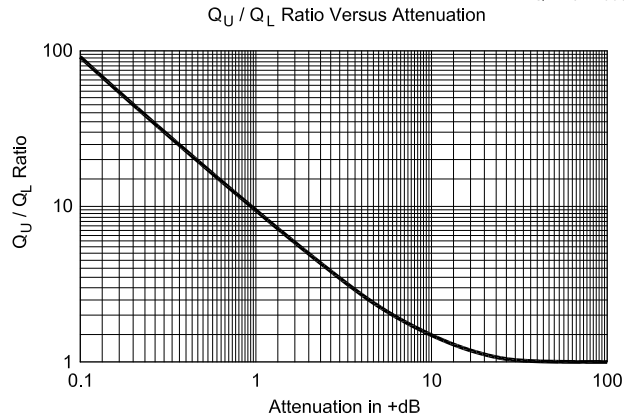


Figure 3 — Graph showing the ratio of unloaded  $Q$  ( $Q_U$ ) to loaded  $Q$  ( $Q_L$ ) as a function of the L-C circuit attenuation at resonance. Applies to a single tuned circuit with identical in / out coupling. When the attenuation is large (> 20 dB) the coupling is minimal and the loaded  $Q$  closely approaches the unloaded  $Q$ .

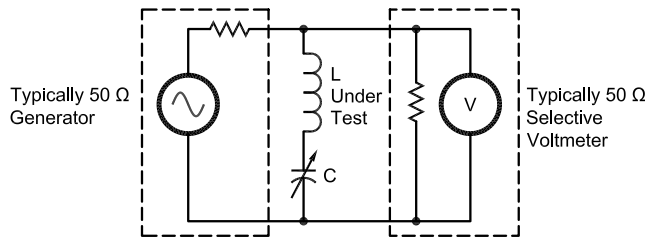


Figure 4 — Here the L-C circuit is connected in series and across the generator - detector. At resonance the L-C circuit has minimum impedance and by measuring the attenuation, the effective series resistance ESR of the L-C series may be computed at the resonant frequency  $f_0$ .

detailed in the reference of Note 2.) It basically computes the effective series resistance (ESR) of the combined L and C, based on the attenuation in dB at resonance, using a selective voltmeter to prevent source harmonics from affecting the measurement. The ESR may be calculated using Equation 6.

$$ESR = \frac{Z}{2 \left( 10^{\frac{dB}{20}} - 1 \right)} \quad [\text{Eq } 6]$$

where  $dB$  is a positive value of attenuation and  $Z$  is the source and detector impedance. Once the  $ESR$  is known, it is only necessary to compute:

$$Q_U = \frac{X_L}{ESR} \quad [\text{Eq } 7]$$

where  $X_L = 2\pi f_0 L$ , and  $L$  has already been measured separately.

Note 3 gives a reference to a spreadsheet that I have developed to eliminate the need to measure  $L$  and  $C$ . Two more attenuation points are required to compute these values.

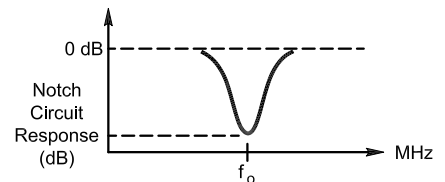


Figure 5 — Frequency response of the notch circuit of Figure 4, where  $f_0$  is the frequency of maximum attenuation.

There is also a version for crystals, which computes the equivalent  $R L C$  values and other related parameters.

## 4 – Reflection Measurement Using an SWR Analyzer

This method has been recently developed to make use of an SWR analyzer, thus eliminating the need for the source-detector combination. Adjust the SWR analyzer to the resonant frequency of the circuit. The L-C circuit may be coupled to a link coil on the

SWR analyzer, which provides variable coupling. As shown in Figure 7, the amount of coupling is adjusted until the SWR drops to 1:1. The frequency is recorded as  $f_0$ . Then the frequency is offset above or below  $f_0$  to obtain an SWR reading between 2 and 5. Now plug the new frequency and SWR values in the spreadsheet that I provide to calculate the unloaded  $Q$  factor. Note that the L or C values are not required to compute the  $Q$  factor.

As shown in Figure 8 the coupling may also be realized with a second variable capacitor,  $C_a$ , which serves as an impedance divider. Both capacitors have their frame grounded, along with the SWR analyzer. This makes the construction and operation of the SWR -  $Q$  meter much easier.

Both circuits transform the L-C circuit effective parallel resistance  $R_p$  to  $50 + j0 \Omega$  as required by the analyzer to obtain a 1:1 SWR at resonance. The capacitance value of  $C_a$  will be in the order of 10 to 50 times the value of  $C$ , the main tuning capacitor. A slight interaction will be present between these two adjustments when adjusting for 1:1 SWR.

Note that a variable inductive divider can be used, in series with the inductor under test, instead of the capacitive divider of Figure 8. In this case the SWR analyzer is connected across this variable inductor, which allows matching to the SWR analyzer  $50 \Omega$  input. I do not recommend this as the  $Q$  of the variable inductor is probably not going to be as high as the matching capacitor,  $C_a$ . The  $Q$  of these matching components have a second order effect on the measured  $Q_U$ .

At a  $Q$  factor of 300, a 1% error in SWR gives a 1% error in  $Q$ . Assume that the measured SWR is 4 and the SWR meter has a resolution of 0.1. If the error in SWR is 0.1, then the error in  $Q$  is 2.5%. Other errors include the  $Q$  of the tuning capacitor, and the  $Q$  of the matching component,  $C_a$ , or the link coil to a lesser degree.

Here are some simulation results:

At  $Q = 200$  the average error on the calculated  $Q$  is  $-0.5\%$  when the test frequency is below resonance. Above resonance, the average error drops to  $-0.05\%$ .

At  $Q = 50$  the average error on the calculated  $Q$  is  $-1.0\%$  when the test frequency is below resonance. Above resonance, the average error drops to  $-0.7\%$ .

These small errors come from the reactance of the coupling element (link coil or  $C_a$  in the capacitive divider).

The following errors were obtained by simulation with an inductor  $Q$  of 200.

The matching capacitor  $C_a$  should have a  $Q$  of 500 or larger. A  $Q$  value of 500 for  $C_a$  gives  $-6\%$  error and a  $Q$  value of 1000 for  $C_a$  gives  $-0.6\%$  error.

Interestingly the link coil  $Q$  may be as low as 50 and the error is only  $-0.4\%$ .

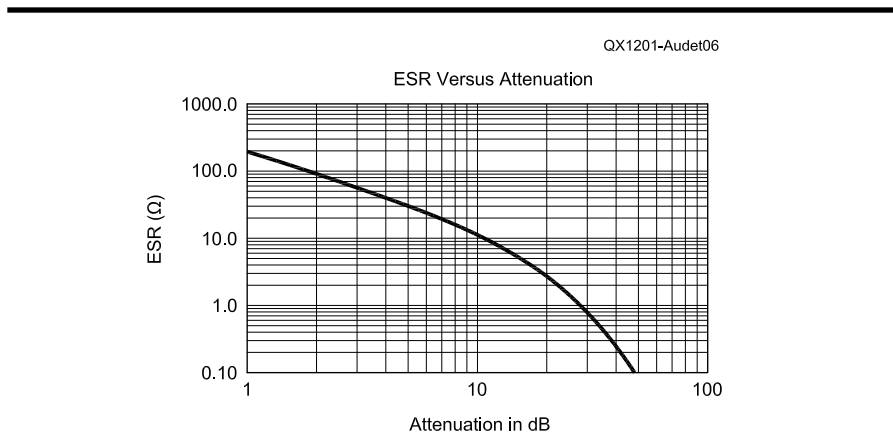


Figure 6 — Effective series resistance as computed from the attenuation in dB, in a  $50 \Omega$  system.

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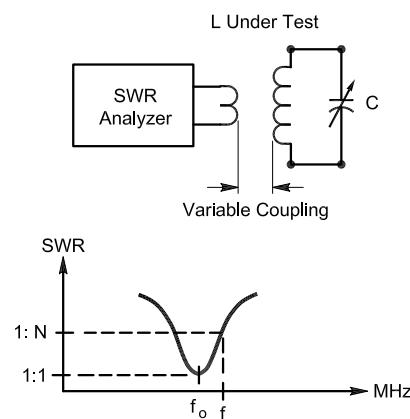


Figure 7 — Set-up for measuring the unloaded  $Q$  of L-C resonant circuit. The coupling is adjusted for 1:1 SWR at resonance. The link should have approximately 1 turn per 5 or 10 turns of the inductor under test.

The losses of the transmission line between the SWR analyzer and the  $Q$  measuring circuit should be kept to a minimum. An attenuation of 0.05 dB gives  $-1.6\%$  error on the calculated  $Q$ .

So far the  $Q$  factor measurement included both the combined  $Q$ s of the L and C, since it is difficult to separate their individual  $Q$  values. When the capacitor  $Q$  ( $Q_{cap}$ ) is known the inductor  $Q$  ( $Q_{ind}$ ) may be calculated like parallel resistors:

$$\frac{1}{Q_{ind}} = \frac{1}{Q} - \frac{1}{Q_{cap}} \quad [\text{Eq 8}]$$

Note that the combined  $Q$  of the L-C circuit is 17% lower with  $Q_{ind} = 200$  and  $Q_{cap} = 1000$ , compared to having a capacitor with an infinite  $Q$ .

QX1201-Audet08

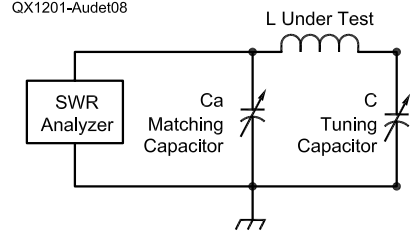


Figure 8 — Here the variable coupling to the SWR analyzer is provided by the capacitive divider formed by  $C_a$  and  $C$ .  $C_a$  is typically 10 to 50 times larger than  $C$ .

### Measuring the $Q$ of Low Value Capacitors ( $< 100 \text{ pF}$ ) Using the Delta $Q$ Method

Two  $Q$  measurements are required:

First measure the  $Q$  of a test inductor (preferably having a stable high  $Q$ ) and record it as  $Q_1$  and record the amount of capacitance  $C_1$  used to resonate it.

Second, connect the low value capacitor to be tested across the tuning capacitor, decrease its capacitance to obtain resonance again. Note the measured  $Q$  as  $Q_2$  and the amount of capacitance  $C_2$  used to resonate. ( $Q_2$  should be less than  $Q_1$ ).

Compute the  $Q$  of this capacitor as follows:

$$Q = \frac{(C_1 - C_2) \times Q_1 \times Q_2}{C_1 \times (Q_1 - Q_2)} \quad [\text{Eq 9}]$$

Note that  $(C_1 - C_2)$  is the capacitance of interest. It may be measured separately using a C meter. If  $Q_2 = Q_1$  the  $Q$  of the capacitor becomes infinite.

### Unloaded and Loaded $Q$

Looking back at method 1, the measured  $Q$  is the loaded  $Q$  and it approaches the unloaded  $Q$  as the source impedance goes toward zero and the detector impedance is infinite.

With method 2, the measured  $Q$  is the loaded  $Q$  and a correction is required based on insertion loss to obtain the unloaded  $Q$ . In this case the L-C circuit under test “sees” the same  $Q$  as we have measured, (if it could look at its environment by looking towards the source and detector).

Such is not the case in methods 3 and 4. In method 3 the L-C circuit will typically “see” a  $50\ \Omega$  source and a  $50\ \Omega$  detector that are effectively in parallel. This adds  $25\ \Omega$  in series with the ESR of the L-C circuit, and reduces the  $Q$  that it sees. This loaded  $Q$  sets the selectivity of the notch filter created by the circuit.

In method 4, the L-C “sees” the SWR analyzer internal impedance, say  $50\ \Omega$ . Remember that the  $R_p$  value is transformed to  $50\ \Omega$  at the analyzer, and effectively is in parallel with the analyzer’s internal impedance. This means that the actual  $Q$  factor seen by the L-C circuit is reduced by half. This value is the loaded  $Q$  of the L-C circuit. In this case the coupling factor equals 1.<sup>5</sup>

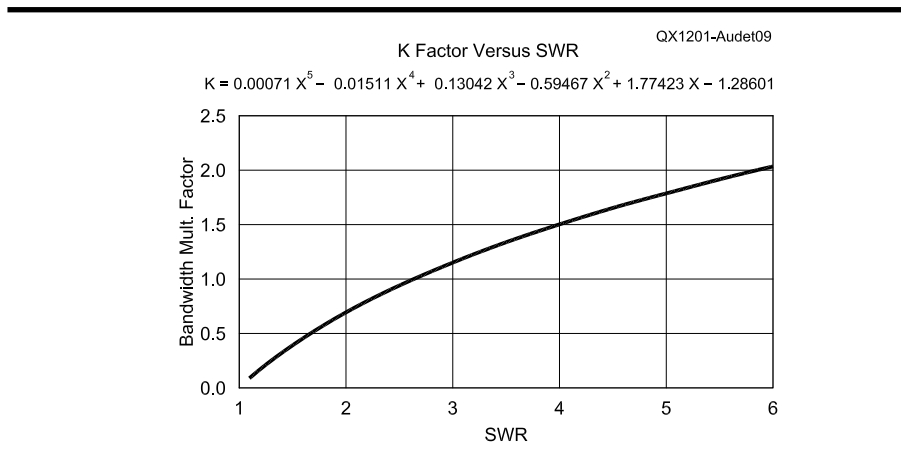
Now consider a  $50\ \Omega$  dipole antenna. The series R-L-C model may be applied to a resonant dipole antenna, which presents approximately  $50\ \Omega$  at the feed point. When the dipole is fed by a low loss transmission line the SWR meter connected at the transmitter will measure the antenna unloaded  $Q$  at an SWR = 2.62 as calculated from equation 3:  $Q = f_0 / BW$ . Note that the spreadsheet given in Note 4 will calculate the dipole  $Q$  factor under these conditions.

When the antenna is fed by a  $50\ \Omega$  source, its effective bandwidth will double, since the total resistance seen by the dipole is now the radiation resistance plus the transmitter output resistance.

Here we still have the loaded  $Q =$  unloaded  $Q / 2$ . This assumes that the transmitter output impedance and the feed line are  $50\ \Omega$ . The  $Q$  derived from the bandwidth at SWR points of 2.62 gives the unloaded  $Q$  value, independently of the transmitter output impedance.

In the general case, the complex impedance of the transmitter reflected at the antenna will modify its effective bandwidth and its actual resonant frequency. Note that the SWR meter connected at the transmitter will *not* show this effect since it can only measure the unloaded  $Q$  of the dipole. Also, the bandwidth of the matching circuits at the transmitter will affect the effective impedance seen by the antenna and modify its loaded  $Q$  and bandwidth.

One way to measure the effective bandwidth of the dipole might be to insert an RF ammeter in series with one dipole leg. Find the frequencies where the current is reduced by  $\sim 30\%$  and compute the effective bandwidth this way. In the receive mode



**Figure 9 — This graph shows the K factor versus SWR. At SWR = 2.62 the correction factor K equals 1 while at SWR = 5.83 the K factor is 2.**

the dipole antenna will also exhibit the same loaded  $Q =$  unloaded  $Q / 2$  if the receiver impedance is  $50\ \Omega$ . Your antenna effective bandwidth may be twice as large as you really thought!

### Summary of the Four Methods Presented in this Article

1 – Classical  $Q$  Meter. See Figure 1. This shows a set-up as used by the HP / Agilent model HP 4342A.

This technique uses a transmission method that requires a very low impedance source and very high impedance detector, which are not easy to realize.

Set the frequency and adjust the reference capacitor for resonance.

The measurement approximates the unloaded  $Q$ . Corrections are difficult to apply.

You don’t have to know the L-C values.

2 – Transmission Method Using Coupling. See Figure 2. This technique requires a source and a voltmeter.

Find the  $-3\ \text{dB}$  transmission bandwidth points.

The measurement requires two low-value, high  $Q$  coupling capacitors.

It measures the loaded  $Q$  as used in the test circuit.

You need to correct the loaded  $Q$  value for attenuation to find the unloaded  $Q$ , if the attenuation is less than 30 dB.

The accuracy of the  $Q$  measurement remains somewhat dependant upon the  $Q$  factor of the coupling capacitors. Refer to Figure 2. With 1 pF coupling capacitors having a  $Q$  of 1000, the error on  $Q$  was  $-1.4\%$ .

The L or C values are not required.

3 – Shunt mode transmission method with LC connected in series. See Figure 4.

This method requires a source and a

selective voltmeter to prevent source harmonics from affecting the measurement.

Compute the ESR from the minimum attenuation measured at resonance.

The L or C values are required to compute the  $Q$  value.

The author’s spreadsheet computes the series/parallel R-L-C values from 3 attenuation measurements. This technique is also useful for crystal measurements.

This is potentially the most accurate method, if the attenuation is measured with high accuracy using a vector network analyzer (VNA).

4 – Reflection measurement using an SWR analyzer. See Figures 7 and 8.

This technique requires a matching capacitor or variable link coupling. There will be a slight interaction between the matching components and the main tuning capacitor.

Adjust two variable capacitors or vary link coupling to obtain an SWR of approximately 1:1

Toroidal inductors are difficult to test with variable link coupling. Use the capacitive divider method instead. With link coupling, keep the link close to the grounded end of the coil, to minimize capacitive coupling. In general air wound coils are more delicate to test since they are sensitive to their environment.

The L and C values are not required.

Offset the frequency to have an SWR increase. Note the frequency and SWR.

The author’s spreadsheet computes the unloaded  $Q$  factor directly.

This technique requires the least equipment of any of the methods.

Best accuracy is obtained with a link coil or a variable capacitive divider.

While all four methods do not require phase or complex impedance measurements,

method 4 only requires an SWR analyzer. The accuracy of the SWR analyzer method has been validated from RF Simulations and by actual measurements.

Since this method relies on SWR measurements, the accuracy of the measuring instrument should be verified. The SWR = 2 point may easily be verified by placing two accurate ( $\leq 1\%$ )  $50\ \Omega$  resistors in parallel at the analyzer output using a tee connector. Note the reading obtained at the frequency of interest and compute the SWR offset from the ideal value of 2.00. Then perform the  $Q$  measurements around this same SWR value, possibly above and below the resonant frequency and average the  $Q$  results.

To compensate for inaccuracies in determining the frequency ( $f_0$ ) of 1:1 SWR, it is recommended that you perform the  $Q$  measurement at two frequencies above and below the  $f_0$ , as done in the spreadsheet provided in Note 4.

Note 5 covers other methods of measuring  $Q$  using a vector network analyzer, mostly suitable for use at microwave frequencies.

*Jacques Audet, VE2AZX, became interested in radio at the age of 14, after playing with crystal radio sets and repairing old receivers. At 17, he obtained his first ham license, and in 1967 he obtained his B Sc degree in electrical engineering from Laval University. He then worked in engineering functions at Nortel Networks, where he retired in 2000. He worked mostly in test engineering on a number of products and components operating from dc to light-wave frequencies.*

*His areas of interest are in RF simulations, filters, duplexers, antennas and using computers to develop new test techniques in measurement and data processing.*

## Notes

<sup>1</sup>Hewlett Packard Journal, September 1970, [www.hpl.hp.com/hpjournals/pdfs/IssuePDFs/1970-09.pdf](http://www.hpl.hp.com/hpjournals/pdfs/IssuePDFs/1970-09.pdf)

<sup>2</sup>Wes Hayward W7ZOI "Two Faces of Q," [w7zoi.net/2faces/twofaces.html](http://w7zoi.net/2faces/twofaces.html)

<sup>3</sup>There is an Excel spreadsheet to perform the various  $Q$  calculations on my website: [ve2azx.net/technical/Calc\\_Series-Par\\_RLC.xls](http://ve2azx.net/technical/Calc_Series-Par_RLC.xls) for RLC circuits.

<sup>4</sup>The MathCad™ and the corresponding Adobe PDF file as well as the Excel file may be downloaded from the ARRL website at [www.arrl.org/qexfiles/](http://www.arrl.org/qexfiles/). Look for the file **1x12\_Audet.zip**.

<sup>5</sup>Darko Kajfez "Q Factor Measurements, Analog and Digital," [www.ee.olemiss.edu/darko/rfqmeas2b.pdf](http://www.ee.olemiss.edu/darko/rfqmeas2b.pdf)

## Appendix 1

In the SWR analyzer method, the equivalent parallel resistance  $R_p$  of the L-C circuit is transformed to  $50\ \Omega$  to give 1:1 SWR at resonance by using an adjustable link coupling or a variable capacitive divider. As the frequency is varied around resonance, it may be set at two frequencies where the absolute value of the reactance,  $X$ , is equal to  $R_p$ .

$$\text{Abs}(X) = R_p$$

Normalizing these impedances gives impedances of 1 for  $R_p$  and  $j$  for  $X$ . Since these are in parallel, the resulting impedance,  $Z$ , will be:

$$Z = \frac{1 \times j}{1 + j} = 0.5 + 0.5j \quad [\text{Eq A-1}]$$

Taking the absolute value of  $Z$  we get  $Z = 0.707$ . This is the  $-3$  dB point, since this value is 3 dB below the initial value of 1.

The complex reflection coefficient is:

$$\rho = \frac{(0.5 + 0.5j) - 1}{(0.5 + 0.5j) + 1} = -0.2 + 0.4j \quad [\text{Eq A-2}]$$

The absolute value of the reflection coefficient  $|\rho|$  is 0.447. The return loss  $RL$  in dB is given by:

$$RL = -20 \log |\rho| = 6.99 \quad [\text{Eq A-3}]$$

The corresponding SWR is given as:

$$SWR = \frac{1 + \rho}{1 - \rho} = 2.62 \quad [\text{Eq A-4}]$$

At the  $-3$  dB points, the bandwidth (BW) is related to the unloaded  $Q$  factor as follows:

$$Q_U = \frac{f_0}{BW}$$

Therefore, measuring the bandwidth,  $BW$ , at  $SWR = 2.62$  allows us to compute the unloaded  $Q$  ( $Q_U$ ) of the circuit. So far, we need three measurement points. One point at frequency  $f_0$  that gives 1:1 SWR and two points at  $f_L$  and  $f_H$  below and above resonance that give an SWR of 2.62. The last two points are symmetrical around  $f_0$ , so only one is measured as  $f = f_H$ .

$$\text{Since } f_L \times f_H = f_0^2 \quad [\text{Eq A-5}]$$

The other frequency  $f_X$  is assumed to be at:

$$f_X = \frac{f_0^2}{f_H} = \frac{f_0^2}{f} \quad [\text{Eq A-6}]$$

The bandwidth  $BW_1$  may now be calculated:

$$BW_1 = f - f_X = f - \frac{f_0^2}{f} \quad [\text{Eq A-7}]$$

Since we want to be able to measure at any SWR value, we need to add a correction factor,  $K$ , resulting from the use of bandwidth  $BW_1$ .

$$Q_U = \frac{K f_0}{BW_1} \quad [\text{Eq A-8}]$$

The correction factor,  $K$ , is the bandwidth multiplication factor. It is a function of the SWR. I derived  $K$  using Mathcad™ numerical calculations and then normalized the value. I used a fifth order polynomial to fit the computed data. See Figure 9. Note 4 gives the Mathcad™ and Adobe PDF files that I used, as well as more details on the derivations. The complete equation as used in my spreadsheet for the unloaded  $Q$  factor becomes:

$$Q_U = \frac{f_0 \cdot (1.77423 \cdot SWR - 0.59467 \cdot SWR^2 + 0.13042 \cdot SWR^3 - 0.0151 \cdot SWR^4 + 0.00071 \cdot SWR^5 - 1.28601)}{\left| f - \frac{f_0^2}{f} \right|} \quad [\text{Eq A-9}]$$

**QEX**