SWR and BANDWIDTH OF SERIES / PARALLEL R L C

1- Series R L C circuit - Calculations and Simulations

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$$Z = j\cdot\omega\cdot L + \frac{1}{j\cdot\omega\cdot C} + R$$ Impedance of a series R L C — Eq 1

$$Q = \frac{\omega\cdot L}{R} \qquad \text{Then :} \quad R = \frac{\omega\cdot L}{Q}$$ Q factors (unloaded !) — Eq 2

$$Q = \frac{1}{\omega_o\cdot C\cdot R} \qquad \text{Then :} \quad R = \frac{1}{\omega_o\cdot C\cdot Q}$$ ω_o is the resonant frequency — Eq 3

The impedance in Eq 1 is normalized

$$\frac{Z}{R} = j\cdot\omega\cdot\frac{L}{R} + \frac{1}{j\cdot\omega\cdot C\cdot R} + 1$$ Eq 4

$$R = \frac{\omega_o\cdot L}{Q} = \frac{1}{\omega_o\cdot C\cdot Q}$$ Eq 5

Substitute Eq 5 into Eq 4:

$$\frac{Z}{R} = j\cdot\omega\cdot\frac{L\cdot Q}{\omega_o\cdot L} + \frac{\omega_o\cdot C\cdot Q}{j\cdot\omega\cdot C} + 1$$ Eq 6

$$\frac{Z}{R} = j\cdot\omega\cdot\frac{Q}{\omega_o} + \frac{\omega_o\cdot Q}{j\cdot\omega} + 1$$ After simplification — Eq 7

$$\frac{Z}{R} = j\cdot Q\cdot\left(\frac{\omega}{\omega_o} - \frac{\omega_o}{\omega}\right) + 1$$ Note that at resonance $\omega = \omega_o$ and Z / R = 1 — Eq 8

At ω_H and ω_L the half power frequencies, the magnitude of Z/R equals $1 + j$
Therefore

$$\left|Q\cdot\left(\frac{\omega_H}{\omega_o} - \frac{\omega_o}{\omega_H}\right)\right| = \left|Q\cdot\left(\frac{\omega_L}{\omega_o} - \frac{\omega_o}{\omega_L}\right)\right| = 1$$ Eq 9

Then

$$\frac{\omega_H}{\omega_o} - \frac{\omega_o}{\omega_H} = \frac{\omega_o}{\omega_L} - \frac{\omega_L}{\omega_o}$$ Eq 10

Rearranging the terms:

$$\frac{\omega_H}{\omega_o} + \frac{\omega_L}{\omega_o} = \frac{\omega_o}{\omega_L} + \frac{\omega_o}{\omega_H}$$ Eq 11

$$\frac{\omega_H + \omega_L}{\omega o} = \frac{\omega o \cdot (\omega_H + \omega_L)}{\omega_H \cdot \omega_L}$$ Eq 12

Therefore

$$\omega_H \cdot \omega_L = \omega o^2$$ Eq 13

$$\omega o = \sqrt{\omega_H \cdot \omega_L}$$ Eq 14

Rearranging the first term of eq 9:

$$Q \cdot \frac{(\omega_H)^2 - \omega o^2}{\omega o \cdot \omega_H} = 1$$ Eq 15

From a combinaison of Eq 13 and Eq 15 :

$$Q \cdot \frac{(\omega_H)^2 - \omega_H \cdot \omega_L}{\omega o \cdot \omega_H} = 1$$ Eq 16

$$Q \cdot \frac{\omega_H - \omega_L}{\omega o} = 1$$ Eq 17

Then

$$\omega_H - \omega_L = \frac{\omega o}{Q}$$ Eq 18

Converting to frequency:

$$f_H - f_L = \frac{f_o}{Q}$$ Eq 19

f_o is the resonant frequency

f_H and f_L are the half power frequencies

The bandwidth BW in Hz is defined as:

$$BW = f_H - f_L$$ Eq 20

Then from Eq. 19

$$BW = \frac{f_o}{Q}$$ And: $$Q = \frac{f_o}{BW}$$ Eq 21a, 21b

Eq 8 repeated

$$\frac{Z}{R} = j \cdot Q \cdot \left(\frac{\omega}{\omega o} - \frac{\omega o}{\omega} \right) + 1$$ Eq 8

For the SERIES RLC circuit :

We have established that at $\omega = \omega_H$ and $\omega = \omega_L$ the **normalized** impedance is:

$\frac{Z}{R} = 1 + j$ or $\frac{Z}{R} = 1 - j$

Note that at resonance Z / R = 1 from Eq. 8.
This is the minimum value.

The reflection coefficient at $\omega = \omega_H$ and $\omega = \omega_L$ is:

These are calculated with respect to the normalized Zo value of 1

$$\text{rho} := \frac{(1+j)-1}{(1+j)+1} \qquad \text{rho} = 0.2 + 0.4i \qquad |\text{rho}| = 0.447$$ **Reflection coefficient**

$$\text{rho} := \frac{(1-j)-1}{(1-j)+1} \qquad \text{rho} = 0.2 - 0.4i \qquad |\text{rho}| = 0.447$$ **Reflection coefficient**

For the PARALLEL RLC circuit :

At $\omega = \omega_H$ and $\omega = \omega_L$ the **normalized** impedance Z/R is **1** in parallel with **+ / - j**:

Combining these impedances in parallel:

$$\frac{Z}{R} = \frac{1 \cdot j}{1 + j} = 0.5 + 0.5i$$

$$\frac{Z}{R} = \frac{1 \cdot (-j)}{1 - j} = 0.5 - 0.5i$$

The reflection coefficient at $\omega = \omega_H$ and $\omega = \omega_L$ is:

$$\text{rho} := \frac{(0.5+0.5i)-1}{(0.5+0.5i)+1} \qquad \text{rho} = -0.2 + 0.4i \qquad |\text{rho}| = 0.447$$ **Reflection coefficient**

$$\text{rho} := \frac{(0.5-0.5i)-1}{(0.5-0.5i)+1} \qquad \text{rho} = -0.2 - 0.4i \qquad |\text{rho}| = 0.447$$ **Reflection coefficient**

For BOTH series and parallel circuits, the reflection coefficients, return loss and SWR all have the same absolute value at the 3 dB points, where $\omega = \omega_H$ and $\omega = \omega_L$

$$\text{RL} := -20 \cdot \log(|\text{rho}|) \qquad \text{RL} = 6.99$$ **Return Loss**

$$\text{SWRt} := \frac{1 + |\text{rho}|}{1 - |\text{rho}|} \qquad \text{SWRt} = 2.61803$$ **SWR Reference Value**

Example of R L C SERIES CIRCUIT

$fr := 10$ MHz $\quad L := 100$ uH $\quad R := 50$ ohms $\quad Zo := R$

$$C := \frac{10^6}{(2\cdot\pi\cdot fr)^2\cdot L}$$

$C = 2.533$ $\quad$ C in pF is calculated to resonate at fr

$XL(f) := j\cdot 2\cdot\pi\cdot f\cdot L$ $\quad$ $XL(fr) = 6.283i \times 10^3$ $\quad$ Reactances

$$XC(f) := \frac{1}{j\cdot 2\cdot\pi\cdot f\cdot C\cdot 10^{-6}}$$

$XC(fr) = -6.283i \times 10^3$

$Z(f) := R + XL(f) + XC(f)$ $\quad$ $Z(fr) = 50$ $\quad$ At resonance

$$rho(f) := \left|\frac{Z(f) - Zo}{Z(f) + Zo}\right|$$

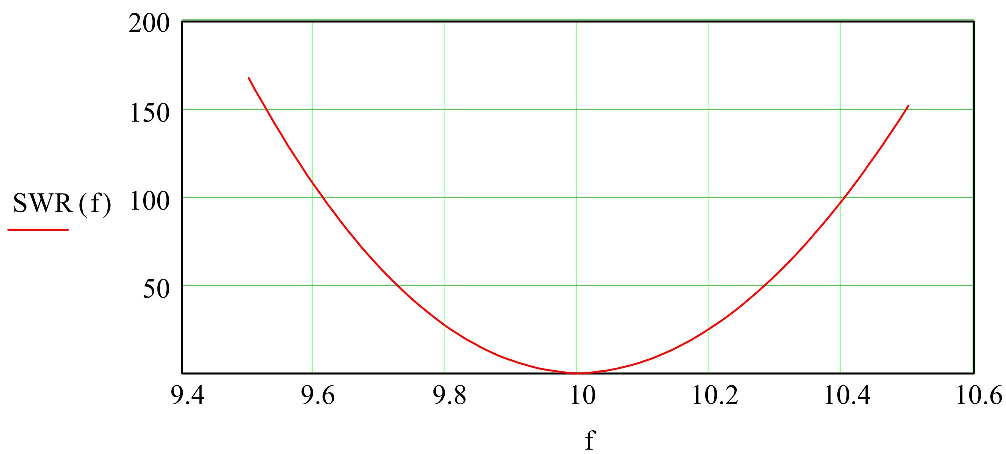
$rho(fr) = 0$ $\quad$ **rho = 0**

$$SWR(f) := \frac{1 + rho(f)}{1 - rho(f)}$$

$SWR(fr) = 1$

The minimum SWR should be 1:1 at fr

$f := 9.5, 9.51 .. 10.5$



Finding the two frequencies F1 and F2 that give SWR = 2.618

$SWRt = 2.618$ target SWR

$Fa := 0.9998 \cdot fr$ start searching for f_low

Given

$SWR(Fa) = SWRt$

$F1(SWRt) := Find(Fa)$ $f\_low := F1(SWRt)$ $f\_low = 9.96029$

$fr - f\_low = 0.0397$

$Fb := 1.0002 \cdot fr$ start searching for f_hi

Given

$SWR(Fb) = SWRt$

$F2(SWRt) := Find(Fb)$ $f\_hi := F2(SWRt)$ $f\_hi = 10.03987$

$f\_hi - fr = 0.0399$

Calculate the Relative Bandwidth Rel_BW (with respect to the resonant freq) at the target SWR

$$Rel\_BW(SWRt) := \frac{F2(SWRt) - F1(SWRt)}{fr} \qquad Rel\_BW(SWRt) = 7.9578 \times 10^{-3}$$

We can calculate the Q at the target SWR = 2.618 (unloaded !)

$$Q := \frac{1}{Rel\_BW(SWRt)} \qquad Q = 125.664$$

Q from the circuit values, we get the same Q value: (unloaded !)

$$Q := \frac{2 \cdot \pi \cdot fr \cdot L}{R} \qquad Q = 125.664$$

$SWRref := 2.618$

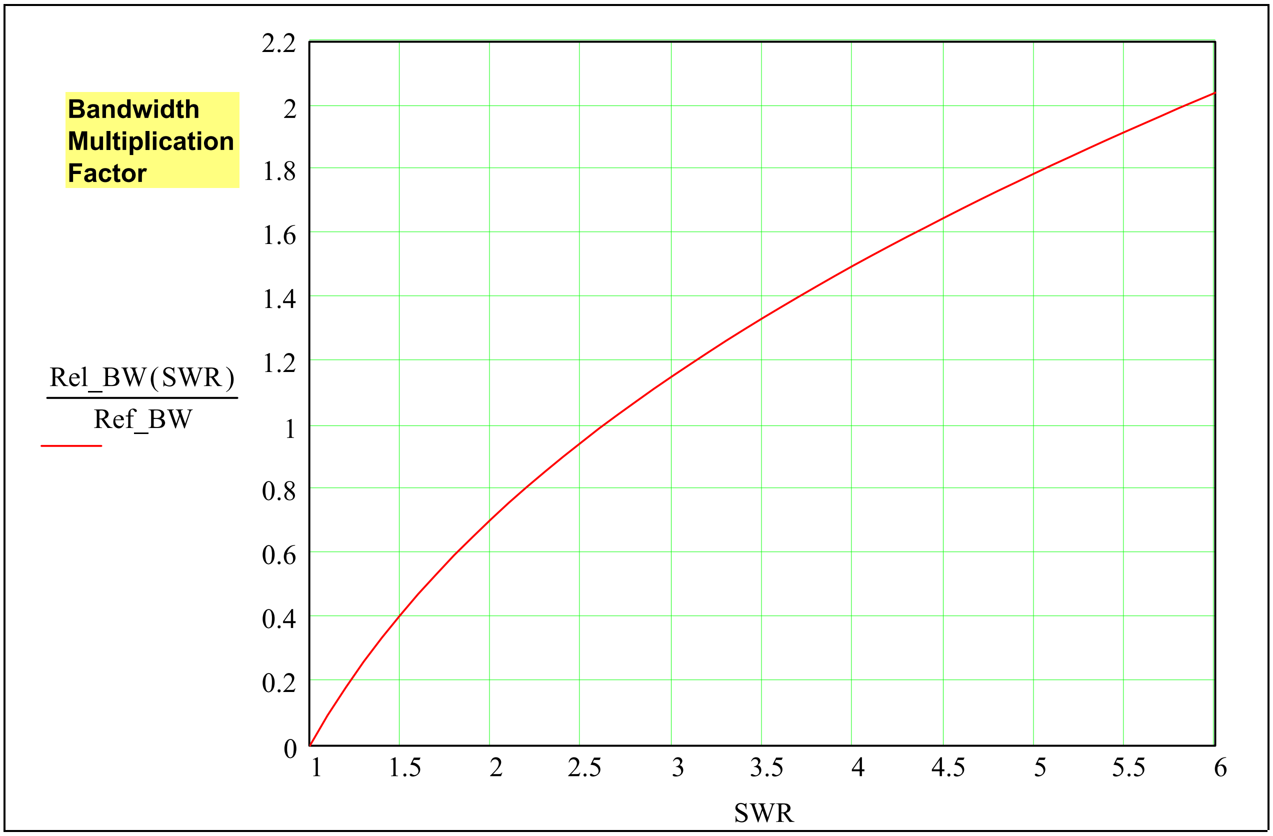
The reference BW is the BW at SWR = 2.618

$$Ref\_BW := Rel\_BW(SWRref) \qquad Ref\_BW = 7.9576 \times 10^{-3}$$

$$\frac{1}{Ref\_BW} = Q$$

Relative BW / Reference BW vs SWR

$SWR := 1, 1.1 .. 6$



For SWR = 2, the BW is 0.707 times the BW at the reference SWR of 2.618

For SWR = 5.83, the BW is 2.0 times the BW at the reference SWR of 2.618

$\frac{Rel\_BW(2)}{Ref\_BW} = 0.7071$ $\frac{Rel\_BW(5.83)}{Ref\_BW} = 2$ $\frac{Rel\_BW(4)}{Ref\_BW} = 1.5$

Pick the SWR value that you like !

For the **series R L C** circuit, a a random length of low loss 50 ohm coax cable may also be connected between the RLC and the SWR measuring instrument.
With a coax line, the impedance will differ from 50 +/- j 50 ohms at the SWR = 2.618 points, leaving the SWR bandwidth unchanged.